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Herd Behavior and Volatility Persistence in Bombay (Mumbai) Stock Exchange

Onour

Herd Behavior and Volatility Persistence in Bombay (Mumbai) Stock Exchange[1]

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Abstract

This paper employs a combination of unit root tests and fractional integration techniques using the ARFIMA(p,d,q) model to test rational bubbles, which implies herd behavior, in Bombay Stock Exchange (BSE). The results in the paper strongly support the evidence of herd behavior in the daily, weekly, and monthly price aggregates. Moreover, the paper also investigates the degree of conditional volatility persistence using FIGARCH(p,d,q) specification to show that the persistence of shocks to stock price and dividend yield volatilities is short-termed.

Keywords: Bubbles; Volatility; GARCH.

1. INTRODUCTION

In the wake of economic liberalization, investors are relying more and more on capital markets, as corporate restructuring (mergers, divestiture, etc.) are becoming commonplace, and strategic alliances are gaining popularity. In these events, a crucial issue to be addressed is: how should a company or a division thereof be valued? In an efficient market, the present value of the expected future dividends of a share represents the fundamental value of the share. This is because in an efficient market stock prices change only in response to new information about the change in fundamentals. When investors purchase shares solely for their future payoff (dividends), stock prices are said to be driven mainly by fundamentals. However, in a market dominated by nonfundamental speculative factors, stock price diverges from its fundamental value. Thus, the systematic divergence of the stock price from its fundamental value is an indication of a rational bubble. Blanchard and Watson (1982) refer to rational bubbles as self-fulfilling expectations that push stock prices toward the expected price level, which is unrelated to change in the fundamentals of the stock price. Bikhchandani and Sharma (2000) attribute rational bubbles to the presence of a large number of investors reacting simultaneously to new information so that an overreaction in aggregate is created. A number of authors (Campbell and Shiller, 1987; Diba and Grossman, 1988; Timmermann, 1995; Nasseh and Strauss, 2004; Koustas and Serletis, 2005) have all investigated the presence of rational bubbles in a number of developed stock markets by investigating integration of stock prices and dividends. The main difference between this paper and the above-mentioned papers is that in this paper our aim to test rational bubbles in a fast-growing major emerging stock market, which is the Bombay stock market (BSE). Along the same lines of the research methodology of Koustas and Serletis (2005), in this

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paper, we employed the fractional integration technique, which uses the more general approach of $I(d)$, where d, is not necessarily equal to 0, or 1, but can take a value of fraction, or $0 < d < 1$. In doing so, we avoid the strong restriction produced by the $I(0)$ and $I(1)$ specification of classical unit root tests.

2. MODELING RATIONAL BUBBLES

In modeling rational bubbles, we adopt the same approach as in Campbell *et al.* (1997). As stock return at time $t + 1$ can be defined as the capital gains plus expected dividend yield, then

$$r_{t+1} = \frac{P_{t+1} - P_t}{P_t} + \frac{D_{t+1}}{P_t}$$

(1)

where r_{t+1} is the return of a stock held at the end of the period $t + 1$, P_t and D_t, respectively, are the stock price and the dividends payable at the end of period t. Taking the mathematical expectation in Equation (1), based on the available information at time t, and rearranging terms, we get

$$P_t = E_t\left(\frac{P_{t+1} + D_{t+1}}{1 + r_{t+1}}\right)$$

(2)

Now solving Equation (2) forward n-periods yield

$$P_t = E_t\left[\sum_{i=1}^{n}\left(\frac{1}{1 + r_{t+i}}\right)^i D_{t+i}\right] + E_t\left[\left(\frac{1}{1 + r_{t+n}}\right)^n P_{t+n}\right]$$

(3)

To solve for a unique solution, we need to assume that in the long term, the last term in Equation (3) diminishes to zero so that[2]

$$E_t\left[\left(\frac{1}{1 + r_{t+n}}\right)^n P_{t+n}\right] \to 0$$
as $n \to \infty$

(4)

Given that the asymptotic assumption stated in Equation (4) is holding, from (3) the fundamental value of the stock is determined as

$$F_t = E_t\left[\sum_{i=1}^{\infty}\left(\frac{1}{1 + r_{t+i}}\right)^i D_{t+i}\right]$$

(5)

When the asymptotic assumption (4) fails to hold, Equation (3) can be restated as follows:

$$P_t = F_t + B_t$$

(6)

where,

$$B_t = E\left(\frac{P_{t+k}}{1 + r_{t+k}}\right) \quad for \quad k = 1, 2....$$

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[2]This is always true for any positive end-period discount rate (i.e., $r_{t+n} > 0$).

The term B_t in Equation (6) is called a rational bubble, because it is consistent with rational expectation and the time path of the expected return. The time-varying expected stock return component in Equation (6) renders Equation (6) into a nonlinear form. To simplify Equation (6) further, suggest a log-linear approximation to Equation (1) so that

$$\hat{r}_{t+1} = \log(P_{t=1} + d_{t+1}) - \log(P_t)$$
$$= \hat{P}_{t+1} - \hat{P}_t + \log(1 + \exp(\hat{d}_{t+1} - \hat{P}_{t+1}))$$

(7)

where

$$\hat{r}_{t+1} = \log(1 + r_{t+1})$$
$$\hat{P}_t = \log(P_t)$$
$$\hat{d}_t = \log(d_t)$$

Equation (7) is a nonlinear function of the log dividend–price ratio. First-order Taylor expansion around the mean reduces Equation (7) to the log-linear approximation:

$$\hat{r}_{t+1} \approx \alpha + \lambda \hat{P}_{t+1} + (1 - \lambda)\hat{d}_{t+1} - \hat{P}_t$$

(8)

where λ and α are parameters. Equation (8) is a linear difference equation for the log stock price. Solving forward and imposing the no bubble assumption $\lim(i \to \infty) \lambda^i p_{t+i} = 0$, we obtain

$$\hat{P}_t = \frac{\alpha}{1 - \lambda} + \sum_{i=0}^{\infty} \lambda^i \left[(1 - \lambda)\hat{d}_{t+1+i} - \hat{r}_{t+1+j} \right]$$

(9)

In a final step, take the mathematical expectation of (9), based on the available information at time t, and solve for the log dividend–price ratio, so that

$$\hat{d}_t - \hat{P}_t = -\frac{\alpha}{1 - \lambda} + E_t \left[\sum_{j=0}^{\infty} \lambda^j [-\Delta \hat{d}_{t+1+j} + \hat{r}_{t+1+j}] \right]$$

(10)

Equation (10) implies that when the dividend growth factor, Δd_t, and the log of stock returns are stationary stochastic processes, the log dividend yield is stationary, and thus no rational bubble is holding. As a result, to test for rational bubbles in dividend yield, we either test for unit roots in the variables on the right-hand side of Equation (10), or alternatively for a unit root in the left-hand-side variable, which is the log dividend yield.

In this paper, we adopt the fractional integration approach to test for stationarity of dividend yield, besides the classical unit root tests.

In the following section, we discuss the ARFIMA(p,d,q) process $y_t (t = 0, \pm 1, ...)$.

2.1. The ARFIMA Process
The ARFIMA(p,d,q) model can be stated as follows:

$$\varphi(L)(1 - L)^d (y_t - \mu) = \theta(L)\varepsilon_t$$

(11)

where

$$\varphi(L) = \sum_{j=1}^{p} \varphi_j L^j, \quad \theta(L) = \sum_{j=1}^{q} \theta_j L^j,$$
$$(1 - L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k - d)L^k}{\Gamma(k + 1)\Gamma(-d)}$$

and L is the lag operator, d is the fractional differencing parameter, all roots of $\phi(L)$ and $\theta(L)$ are assumed to lie outside the unit circle, and ε_t is white noise.

GARCH(p,q) models attempt to account for volatility persistence but have the features that persistence decays relatively fast. However, in some cases, volatility shows very long temporal dependence, that is, the autocorrelation function decays very slowly. This motivates consideration of fractionally integrated generalized autoregressive conditional heteroskedasticity (FIGARCH) process (Baillie, 1996) defined as follows[3]:

$$\phi(L)(1-L)^d \varepsilon_t^2 = w + \{1 - \beta(L)\}v_t \tag{12}$$

where $\varphi(L)$ *and* $\beta(L)$ are, respectively, the AR(p) and MA(q) vector coefficients and $v_t = \varepsilon_t^2 - \sigma_t^2$,

Following Baillie (1996), Bollerslev and Mikkelsen (1996), and Granger and Ding (1996), the parameters in the ARFIMA(p,d,q) and FIGARCH(p,d,q) models in (11) and (12) were estimated using quasi-maximum likelihood (QMLE) method. In the ARFIMA models, the short-run behavior of the data series is represented by the conventional Auto-Regressive Moving Average (ARMA) parameters, while the long-run dependence can be captured by the fractional differencing parameter, d. A similar result may also hold true when modeling conditional variance, as in Equation (12). Although for the covariance stationary GARCH(p,q) model a shock to the forecast of the future conditional variance dies out at an exponential rate, for the FIGARCH(p,d,q) model the effect of a shock to the future conditional variance decays at low hyperbolic rate. As a result, the fractional differencing parameter, d, in Equations (11) and (12), can be regarded the decay rate of a shock to the conditional variance (Bollerslev and Mikkelsen, 1996).

In general, allowing for values of d in the range between zero and unity (or, $0 < d < 1$) adds flexibility that plays an important role in modeling long-run dependence in time series.[4]

3. TESTING FOR FRACTIONAL INTEGRATION

For computational simplification, the polynomial in (11) and (12) can be presented in a form of binomial expansion as follows:

$$(1-L)^d = \sum_{j=0}^{\infty} \binom{d}{j}(-1)^j L^j = 1 - dL + \frac{d(d-1)L^2}{2} - \ldots..$$

for all real d. If $d = 0$, the series is covariance stationary and possesses short memory process, whereas in the case of $d = 1$ the series is nonstationary. However, in the case of $0 < d < 0.5$, the series even though covariance stationary, its auto-covariance decays much more slowly than the ARMA process. If d is $0.5 < d < 1$ the series is no longer covariance stationary, but still is mean reverting with the effect of a shock persisting for a long period of time, and in that case the process is said to have a long memory. More specifically, given a discrete time series, y_t, with autocorrelation function, ρ_j, at lag j, Mcleod and Hipel (1978) define long memory as a process

$$\sum_{j=-n}^{n} |\rho_j| \qquad as \quad n \to \infty \tag{13}$$

characterized as nonfinite. In the nonstationary and in the long memory process, a shock e_t at time t continues to influence future y_{t+k} for a longer horizon, k, than would be the case for the standard stationary ARMA

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[3]For the FIGARCH(p,d,q) model to be well defined, and the conditional variance positive for all t, all the coefficients in the ARCH representation must be non-negative.
[4]A detailed discussion about the importance of allowing for noninteger values of integration when modeling long-run dependence in the conditional mean of time series data.

process. Although there are varieties of ways to check for the nonstationary long-memory process, in this paper we employed the maximum likelihood estimator.

4.   RESULTS AND DISCUSSION

In this paper, we test the BSE index and its corresponding dividend yield, for the existence of rational bubbles or mean reversion in the log price level and dividend yield using daily, weekly, and monthly data during the period from September 1, 2010 to September 1, 2018. The data in this research are taken from the website of BSE. For the weekly data, we took the averages of the five trading days each week, whereas for the monthly data the average of all trading days in each month is taken as the monthly index. An advantage of taking the average of trading days each week and each month is that it neutralizes the effect of the end of the week and the end of the month on the stock price data. To test for stationarity, we employed traditional Auto Correlation Function (ACF) plotting and unit root tests, the *Augmented* Dickey and Fuller (1981) or ADF, Phillips and Perron (1988) or PP, and Kwiatkowski *et al*. (1992) or KPSS tests, with trend and with no trend in the variables. As indicated in Table 1, and in the ACF plots (figures 1 and 2) the results of the three tests

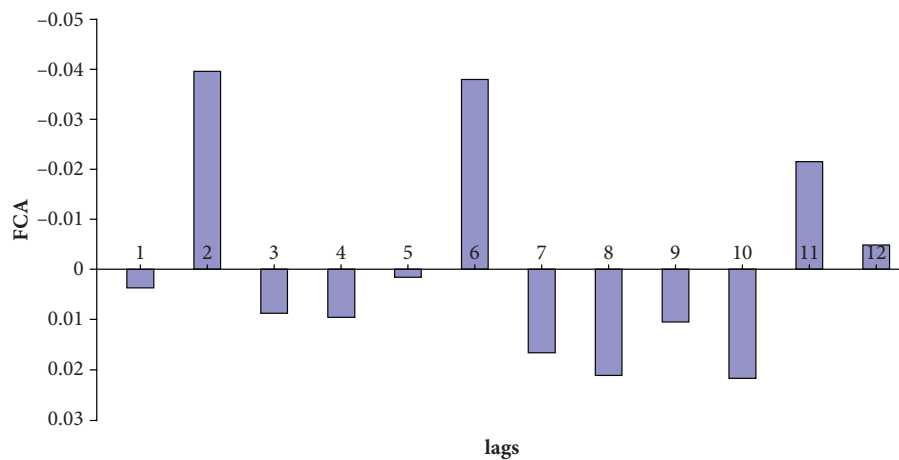**Figure 1.   The Sample ACF of Dividend Yield.**

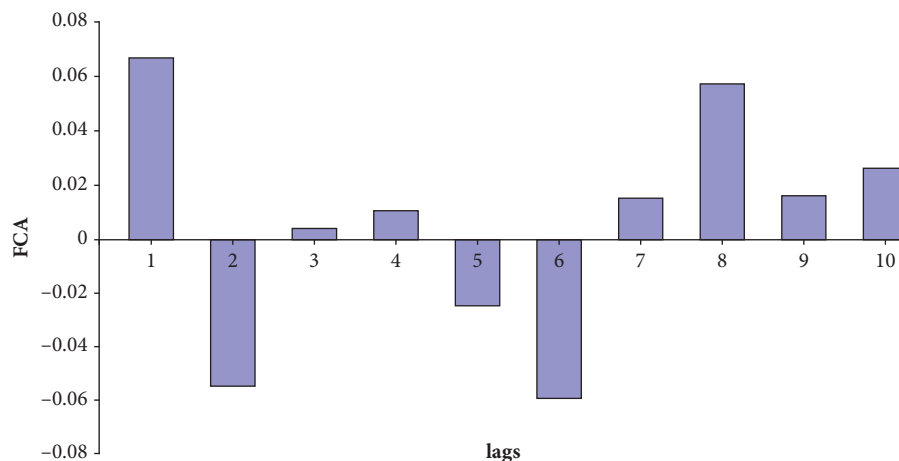**Figure 2.   The Sample ACF of Stock Price.**

Table 1.   Unit Root Tests.

	Dickey–Fuller		Phillips–Perron		KPSS	
	(i)	(ii)	(i)	(ii)	η_u	η_r
Log price:						
Daily data	2.90	3.26	3.10	4.12	3.89	13.31
Weekly data	1.89	3.45	3.89	3.19	9.15	14.40
Monthly data	3.19	4.09	4.01	4.12	1.29	8.09
Log dividends yield:						
Daily data	0.89	2.78	2.41	2.24	25.23	2.51
Weekly data	1.75	1.90	1.31	2.11	15.09	1.67
Monthly data	3.48	3.61	3.82	3.56	1.10	2.10
Critical values (5%) Significance level	4.59	4.68	4.59	4.68	0.463	0.146

Note: (i) with drift only, (ii) with drift and trend. η_u and η_r statistics are, respectively, level stationarity and trend stationarity statistics. The reported KPSS statistics are based on 20 lags for daily, 8 lags for weekly, and 2 lags for monthly data. The optimal lag length order in ADF is selected by Akaike Information Criteria (AIC).

cannot reject the unit root in any of the cases, thereby suggesting the existence of a rational bubble in the BSE price data. However, it is well documented in the literature that the ADF and PP unit root tests, in particular, suffer from very low power against stationary alternatives if the roots are close to the unit root. Diebold and Rudebusch (1991) indicate that ADF and PP unit root tests have very low power against the fractionally integrated alternative. To account for such a shortfall, we investigate the order of integration of the two data sets using the fractionally integrated ARMA process.

Table 1 reports the unit root test results, using *Augmented* Dickey and Fuller (1981), Phillips and Perron (1988), and Kwiatkowski *et al.*, 1992 (KPSS) tests.[5] All the three test results, under a 5% significance level, reject the stationarity condition in favor of the unit root hypothesis, for the daily, weekly, and the monthly log price and the log dividend yield. As indicated earlier, the evidence of unit root (nonstationarity) in the log dividend yield is consistent with the existence of rational bubbles, which imply a persistent deviation of stock prices from its fundamental driver, which is the dividend per share value. As the ADF, PP, and KPSS unit root test results presented in Table 1 reflect only the integer order of integration, the ARFIMA(p,d,q) process can verify the order of integration of the fractional exponent. Thus, the unit root tests reported in Table 1 impose the restriction that $d = 1$, while the KPSS test imposes the restriction of the null hypothesis that $d = 0$, whereas the ARFIMA process reported in Table 2 tests the order of integration under the null hypothesis that $d < 1$. The results reported in Table 2 reject the fractional integration of the log dividend yield and the log price level, for daily, weekly, and monthly time series data. The estimated values of d are significantly greater than the stationary range of ($-0.5 < d < 0.5$). The results of unit root tests and fractional integration test in Tables 1 and 2 both suggest rejection of mean reversion hypothesis in the log prices and log dividend yield, in favor of the unit root hypothesis, which imply rational bubbles behavior in BSE.

The effect of aggregation bias in the data is realized by a number of authors in the literature (Ng and Perron, 1995; Taylor, 2001) and pointed out that the use of low-frequency data increase bias toward random walk process. For instance, Taylor (2001) concludes that if stock price adjustment toward its fundamental value (dividends) is of the order of days or weeks, then using monthly data could bias the results toward finding unit roots in the data, thereby concluding the existence of rational bubbles. To safeguard against these types of aggregation bias, we conducted a Monte Carlo simulation of 2000 replication assuming

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[5]KPSS test initially was developed to test the null hypothesis $I(0)$, against the alternative $I(1)$. However, Lee and Schmidt (1996) indicated (Theorem 3, page 291) that the KPSS test is consistent with the null hypothesis of short memory, against stationary long memory alternatives, such as $I(d)$ process for $d \in (-0.5, 0.5), d \neq 0$. Thus, KPSS test can also be used to distinguish short-memory and long-memory stationary processes.

<div align="center">Table 2.   Estimation Results of ARFIMA(1,<i>d</i>,1).</div>

Parameters	Log dividends yield			Log price level		
	Daily	Weekly	Monthly	Daily	Weekly	Monthly
$\hat{d}$ (std.error)	0.82* (0.15E-3)	0.86* (0.001)	0.79* (0.002)	0.89* (0.16E-3)	0.87* (0.15E-3)	0.91* (0.002)
$\hat{\phi}$ (std.error)	0.001 (0.001)	0.19* (0.001)	0.61* (0.002)	0.046* (0.003)	0.36* (0.16E-2)	0.43* (0.071)
$\hat{\theta}$ (std.error)	−0.12E-5 (0.47E-6)	−0.11E-7* (0.11E-6)	0.12E-5* (0.23E-7)	0.01E-7 (0.19E-6)	0.56E-8 (0.18E-7)	0.1E-7 (0.12E-5)
c (std.error)	0.011* (0.55E-4)	0.0056* (0.0005)	0.018* (0.0021)	0.0078* (0.0001)	0.021* (0.15E-2)	0.10* (0.035)
Log-likelihood function	5,432	2,130	117	1,521	1,130	320

<div align="center">Table 3.   Monte Carlo Simulation.</div>

	Log dividend yield	Log price
Daily		
$\bar{d}$ ESE STDSE	0.82 0.01 0.005	0.89 0.001 0.0003
Weekly		
$\bar{d}$ ESE STDSE	0.86 0.10 0.002	0.90 0.01 0.007
Monthly		
$\bar{d}$ ESE STDSE	0.74 0.17 0.06	0.88 0.11 0.09

Note: $\bar{d}$ is the average parameter estimate. ESE is the average standard error, STDSE is the standard deviation of the standard error.
We used the Data Generating Process (DGP) process of ARFIMA(0,<i>d</i>,1):
$(1 - L)^d (y_t - u) = e_t$ where $e_t = \theta e_{t-1} + \varepsilon_t$ for ε_t is white noise

random walk Data Generating Process. The simulation results in Table 3 show the fractional difference parameter, <i>d</i>, is unbiased and therefore complement the significance of the results in Table 3, that is the unit root hypothesis of both log dividend yield and the log price level.

    Table 4 presents the results of the volatility persistence of the FIGARCH model. The sign and size of the $\hat{d}$ parameter in the FIGARCH model indicate that there is no evidence of long-memory behavior in the conditional variance of the dividend yield and the stock price.

5.   CONCLUSION

This paper has employed a combination of unit root tests and fractional integration techniques to test the order of integration of log dividend yield and log price level in BSE. The paper also investigates the degree

Table 4. FIGARCH(1,*d*,1).

Parameters	Log dividends yield			Log price level		
	Daily	Weekly	Monthly	Daily	Weekly	Monthly
$\hat{d}_1$ (std.error)	0.072* (0.031)	0.11* (0.01)	0.10* (0.10)	0.45* (0.010)	−0.41* (0.021)	0.30* (0.11)
$\hat{\phi}_1$ (std.error)	−0.010* (0.013)	−0.026 (0.10)	−0.06* (0.19)	−0.12* (0.026)	0.68* (0.012)	−0.15 (0.19)
$\hat{\theta}_1$ (std.error)	0.046 (0.87)	0.010 (0.80)	0.020 (0.72.)	0.059 (0.34)	0.09 (0.67)	0.01 (0.90)
Log-likelihood function	465	215	112	465	813	430

of conditional volatility persistence using the FIGARCH(p,d,q) model for the log dividend and the log price data on daily, weekly, and monthly series, for the period from September 1, 2010 to September 1, 2018. The results in the paper strongly support the existence of rational bubbles in the daily, weekly, and monthly aggregates. The Monte Carlo simulation results fully support our estimation results and show no aggregation bias effect on the results. The evidence of rational bubbles in BSE reflects the consistent divergence of stock prices from dividends. The presence of rational bubbles in BSE can be viewed as an indication of herd behavior in the market, as a large number of investors reacting simultaneously to new information create overreaction in aggregate.

Conflict of Interest
None.

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