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Unraveling The Dynamics Of Foreign Direct Investment And Economic Activity In Brazil: A Copula-Based Analysis

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ABSTRACT
This study analyzes the relationship between FDI and the IBC-Br from 2003 to 2022. Our objective is to examine the effect of FDI on Brazil´s economic activities. Thus, we also look for a connection between the Brazilian Central Bank’s leadership in each period and the FDI effect on IBC-Br. To explore this, we employ an approach based on the Granger Causality Test and copula models that encompass Gaussian, t-Student, Gumbel, Frank, and Clayton functions. Our findings indicate that in the short term, FDI exerts some influence on IBC-Br, while the reverse relationship is not observed. Furthermore, using the Clayton Copula function, we identify the dependency of causal relationships between the variables, particularly in extreme economic scenarios. These discoveries contribute to a more comprehensive understanding of the dynamics between FDI and economic activity in the different periods of the Brazilian Central Bank’s leadership.

KEYWORDS: Foreign Direct Investment; Economic Activity; Copula.

ABBREVIATIONS
FDI: Foreign Direct Investment; IBC-Br: Economic Activity Index of Brazil;

1. INTRODUCTION

In recent decades, FDI has surged dramatically, with Brazil’s prominence in the global economic market. Notably, in 2022, Brazil retained its position as the sixth-largest recipient of FDI, attracting a substantial inflow of $86 billion during that year. Additionally, on a global scale, 70% of FDI was channeled into developing countries. Latin America and the Caribbean experienced a remarkable 51% increase in FDI inflows from 2021 to 2022 [1].

The substantial increase in FDI in Latin America and the Caribbean was a direct result of the forces of globalization, compelling companies to actively seek this type of investment with the aspiration of being part of economic growth elsewhere. FDI has emerged as a crucial instrument in the development strategies of numerous countries, primarily because of its stability and cost compared with other sources of investment. As a result, endogenous growth models have begun to reveal a range of positive impacts associated with FDI in recipient economies, from an increase in productive capacity to intensified commercial and employment activities [2,3].

However, it is noteworthy that while numerous studies suggest a positive association between FDI and economic growth, a consensus on this issue remains elusive. Studies such as Wang [4] demonstrate that the effects of FDI can be heterogeneous, depending on the sector in which the investment is made. For example, while FDI in the manufacturing sector has a significantly positive impact on economic growth, investment in non-manufacturing sectors does not show the same effect. Additionally, Alvarado *et al*. [5] found that the impact of FDI varies according to the level of development of the countries, being positive and significant in high-income countries, mixed in upper-middle-income countries, and negative in lower-middle-income countries.

The relationship between FDI and economic growth also depends on specific contextual factors such as trade openness, the level of human capital, and the development of the financial market [6,7]. These studies show that the interaction between FDI and these variables often reveals more pronounced effects of FDI on economic growth, highlighting the importance of a favorable context for maximizing the benefits of foreign investment.

Therefore, this article aims to (i) assess the dependency of FDI on Brazilian economic activity using the IBC-Br from the Central Bank of Brazil (BCB) for the period from 2003 to 2022, (ii) identify and characterize the central economic dynamics of the Central Bank's presidents during each of these periods, and (iii) investigate the causal relationship between FDI and economic activity as a whole.

To achieve these objectives, Section 2 provides a comprehensive literature review, and Section 3 outlines the contributions of our study. In Section 4, we present the methodology behind the Granger causality test and the copula model. Section 5 presents a descriptive analysis of the data, while Section 6 presents our findings, indicating the significant influence of FDI on Brazilian economic activity in the short term, particularly during times of economic uncertainty and crisis. Finally, in Section 7, we provide a conclusion, summarizing the key insights gained from our study.

2. THE RELATIONSHIP BETWEEN FDI AND ECONOMIC GROWTH

The relationship between FDI and economic growth has been widely studied over the past decades. Stephen Hymer's initial theories in the 1970s suggest that multinational companies invest in other countries to exploit specific competitive advantages, such as intangible assets and superior competencies [8]. In the 1980s, John Dunning proposed the "Eclectic Paradigm" (Ownership, Location, and Internalization), explaining that FDI occurs due to ownership, location, and internalization advantages [9].

In the 1990s, empirical studies such as those by Borensztein *et al*. [2] indicated that FDI can boost economic growth in developing countries, especially those with a minimum level of human capital. Alfaro *et al*. [3] highlighted the importance of a well-developed financial sector for the full realization of FDI benefits, maximizing the positive spillover effects.

Recent studies have explored the contextual variations in the impact of FDI in different countries and regions. Basu *et al*. [6] showed a bidirectional relationship between FDI and growth in developing countries. Mehic *et al*. [7] found a positive effect of FDI on economic growth in Southeastern European countries. Wang [4] observed that the impact of FDI varies by sector and is significant in the manufacturing sector of Asian economies.

In Latin America, Alvarado *et al*. [5] found that the effect of FDI on economic growth is not statistically significant in aggregate but varies with the development level of the country. In Brazil, studies by Angelo *et al*. [10] and Lima Jr. and Jayme Jr. [11] analyzed the determinants and impacts of FDI, highlighting factors such as consumer market growth and the concentration of investments in mergers and acquisitions. Shahzad *et al*. [12] showed that FDI and domestic savings have insignificant responses to economic growth in Brazil, while remittances of foreign currency and capital formation contribute positively.

The reviewed literature demonstrates that FDI can play a significant role in economic growth, depending on factors such as companies' competitive advantages, favorable conditions in the host country, the level of human capital, and the development of the financial sector. These theoretical and empirical contributions provide a solid foundation for understanding the complex mechanisms by which FDI can influence economic development.

3. CONTRIBUTIONS

This study aims to contribute to the field of economic analysis by first segmenting the analysis of FDI in relation to the Brazilian IBC-Br based on the mandates of the presidents of the Central Bank of Brazil and provides a detailed view of how different political contexts and significant events, such as global economic crises and pandemics, have influenced this relationship over time. The use of advanced statistical methods, such as Granger causality and copulas, enables a robust and sophisticated analysis of the interdependence between FDI and the IBC-Br, providing insights into both short- and long-term dynamics.

Additionally, assessing the variability of time series through log returns offers a detailed understanding of the percentage changes over time. This approach identifies periods of high and low confidence among foreign investors in the Brazilian economy as well as moments of greater or lesser economic activity. The identification of significant causal relationships, especially in short-term periods, highlights that FDI can influence IBC-Br but not vice versa, which is a crucial contribution to future economic policy formulation.

Finally, this study documents how events such as the 2008 financial crisis, the European sovereign debt crisis, the 2016 impeachment, and the COVID-19 pandemic affected the relationship between FDI and the IBC-Br. The analysis revealed the dependency dynamics between the variables, showing how this relationship can substantially change in response to significant events. These findings suggest that adaptive and resilient economic policies are essential to maintaining stability and fostering economic growth in Brazil.

4. METHODOLOGY

First, it is important to highlight that in Brazil, FDI is predominantly directed towards financial services and auxiliary activities. In 2022, these sectors represented 59.5% of the FDI position in equity participation. Other notable sectors included oil and natural gas extraction (12.6%), trade, except vehicles (4.2%), metallic minerals extraction (3.9%), and food products (3.3%). The remainder was distributed among various other sectors [13].

On the other hand, the Central Bank's IBC-Br is an important measure that aims to anticipate the evolution of Brazilian economic activity. Used as a leading indicator of GDP, the IBC-Br provides a comprehensive view of the national economy by aggregating information from various sectors, such as industry, commerce, services, and agriculture. It is a useful tool for economic policy formulation and decisions by investors and economic analysts [14].

Monthly data were collected from the Central Bank of Brazil [15], covering the period from February 2003 to December 2022. Granger causality methods and copula structural dependency functions (Gaussian, t-Student, Gumbel, Frank, and Clayton) were used to understand the relationship of the variables statically and dynamically (60-month windows with 6-month steps). The following subsections present the mathematics of the methods and descriptive data analysis.

Copulas perform a crucial role in analyzing the dependence between economic or financial variables, especially when the dependence is asymmetric or non-linear. They allow for modeling the dependence structure separately from the marginal distributions, providing greater flexibility in analyzing causal relationships. For instance, in the non-linear Granger causality test proposed by Kim *et al*. [16], copulas are used to capture the directional non-linear dependence between financial time series. Similarly, Lee and Yang [17] utilize copulas to examine Granger causality in conditional quantiles, allowing for the prediction of causal relationships at different points of the conditional distribution between financial markets.

The choice of a 60-month window and 6-month intervals was made based on an attempt to capture significant economic variations without losing the ability to detect long-term trends. The literature suggests that a reasonable amount of data is necessary to effectively use copulas, ensuring an adequate capture of the dependence between economic variables. For example, Genest *et al*. [18] discussed the importance of a sufficiently large sample size to accurately estimate copulas. They emphasized that larger datasets help distinguish between different copula models.

Analyzing the variables at this level is essential to understanding the absolute behavior of economic indicators over time, allowing for the observation of long-term trends and the direct impact of economic policies. For FDI, analyzing the accumulated values in millions of dollars provides a clear view of the total volume of foreign investments received by Brazil, while the IBC-Br in absolute values allows for direct comparisons between different periods.

Furthermore, the use of log-return variables is important to capture the percentage variation in financial and economic data. Log-return stabilizes variance over time and facilitates the application of statistical techniques. In the context of this study, the log returns of IBC-Br and FDI help identify periods of greater instability or rapid change, offering a dynamic perspective on the relationship between foreign investment and economic activity.

4.1 GRANGER CAUSALITY

The Granger causality test is a statistical technique designed to assess whether one time series adds meaningful predictive information. Building on the works of Granger [19] and Hamilton [20], the test typically involves two time series, denoted as Y (the potentially causal series) and X (the series under investigation for its relationship with Y). The test formulated the following two key hypotheses:

i) Null Hypothesis (H0): This hypothesis posits that time series Y does not have a Granger causal effect on time series X.

ii) Alternative Hypothesis (H1): This hypothesis asserts that time series Y has a Granger-causal influence on time series X.

The test statistic relies on fitting autoregressive (AR) models of a specified order, typically denoted as "p," to both series Y and X. It then assesses the predictive quality of these models by comparing them as follows:

i) Restricted Model (H0): In this model, only the past lags of X are used to predict the current value of X. This is typically represented by Equation 1, where Y's past lags of Y are not considered in the prediction.

ii) Unrestricted Model (H1): In this model, both the past lags of X and the past lags of Y are employed to predict the current value of X. This model explores the potential causal relationship between Y and X by allowing Y's past lags to influence predictions.

$$X_t = \alpha + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \ldots + \beta_p X_{t-p} + \varepsilon_t \tag{1}$$

$$X_t = \alpha + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \ldots + \beta_p X_{t-p} + \gamma_1 Y_{t-1} + \gamma_2 Y_{t-2} + \ldots + \gamma_q Y_{t-q} + \varepsilon_t \tag{2}$$

To execute the Granger causality test, we computed the F-statistic ratio, which serves as a basis for comparing the predictive performance of the two models. When the calculated F-statistic exceeded the critical value from the F-distribution table for a chosen significance level, we rejected the null hypothesis (H0) and inferred the presence of Granger causality from Y to X. In essence, the Granger causality test methodically evaluates whether a time series enhances predictability in another time series by measuring the improvement in prediction accuracy resulting from the inclusion of lagged values from the first series in modeling the second series. This rigorous procedure helps us ascertain the causal relationships between time series variables.

4.2 COPULAS SPECIFICATION

According to Sklar [21], Nelsen [22], Joe [23], Yamaka and Maneejuk [24], the equations below represent different formulations of the copula function between two variables (u,v), where in this case, (u) represents the deflated and 12-month accumulated FDI, and (v) represents the IBC-Br. Each equation provides a unique mathematical expression for calculating the correlation between these two variables with different parameters and approaches. In Equation 3 (Gaussian Copula), the correlation between u and v is calculated using the joint distribution function Φ_2 and its inverse functions $\Phi^{-1}(u)$ e $\Phi^{-1}(v)$. The parameter ρ controls the intensity of the correlation. Equation 3.1 allows us to calculate Kendall's τ from the parameter ρ, providing insight into the non-linear dependence between the variables represented by the copula.

Kendall's τ is an important measure because it provides a way to assess the correlation between pairs of observations, defined as the difference between the probability that two random variables are concordant and the probability that they are discordant. It ranges from -1 to 1, where positive values indicate concordance and negative values indicate discordance. In terms of copulas, this measure is particularly useful because it allows for the parameterization of the dependence between variables in a nonlinear manner and can be calculated based on copulas. Many properties and measures of dependence, such as Kendall's τ, are "scale-invariant," meaning they remain unchanged under strictly increasing transformations of the random variables. This makes it a robust tool for modeling dependence, as it captures the joint distribution properties that are invariant under such transformations [22].

$$C(u,v) = \Phi_2(\Phi^{-1}(u), \Phi^{-1}(v); \rho) \tag{3}$$

$$\tau = \frac{2}{\pi} arcsin(\rho) \tag{3.1}$$

$$C(u,v;df,\rho) = T_2(T^{-1}(u;df), T^{-1}(v;df); \rho, df) \tag{4}$$

$$\tau = \frac{2}{\pi} arcsin(\rho) \tag{4.1}$$

$$C(u,v;\theta) = \left(u^{-\theta} + v^{-\theta} - 1\right)^{(-1/\theta)} \tag{5}$$

$$\tau = \frac{\theta}{\theta + 2} \tag{5.1}$$

$$C(u,v;\theta) = exp[-((-ln(u))^{\theta} \ + \ (-ln(v))^{\theta})^{(1/\theta)}] \tag{6}$$

$$\tau = 1 - \theta^{-1} \tag{6.1}$$

$$C(u,v;\theta) = -1/\theta \, ln\left[\frac{1 + \left(e^{(-\theta u)} - 1\right)\left(e^{(-\theta v)} - 1\right)}{(e^{(-\theta)} - 1)}\right] \tag{7}$$

$$\tau = 1 + 4\left[\frac{D_1(\theta) - 1}{\theta}\right] \tag{7.1}$$

$$D_1(\theta) = \frac{1}{\theta}\int_0^{\theta} \frac{t}{exp(t) - 1} dt \tag{7.2}$$

Equation 4 (Student's t-Copula) introduces two parameters, df (degrees of freedom) and ρ, to calculate the correlation between (u) and (v). This is done using the multivariate t-distribution function T_2 and its inverse functions $T^{-1}(u; df)$ e $T^{-1}(v; df)$. Equation 4.1 establishes the relationship between Kendall's τ and ρ: Equation 5 (Clayton Copula) calculates the correlation between u and v based on the parameter θ, while Equation 5.1 is used to calculate Kendall's τ. The expression involves θ powers applied to variables u and v. Parameter θ affects the shape of the correlation, making it sensitive to changes in u and v: Equations 6 and 7 present formulas for the Gumbel and Frank copulas, respectively. In the Gumbel copula, parameter θ governs the dependence, and the relationship between Kendall's τ and θ is given by equation 6.1. For the Frank copula, parameter θ also determines the dependence, with Kendall's τ expressed through Equations 7.1 and 7.2; here, $D_1(\theta)$ represents a defined integral that captures the dependency structure.

The choice of multiple copula specifications is based on the need to capture different forms of dependency between the analyzed variables. Each copula offers a distinct way of modeling dependency, with some being more suitable for capturing strong tail dependencies (such as the Gumbel copula), whereas others are more flexible in terms of symmetry (such as the Gaussian copula). The diversity of specifications allows for a more comprehensive and robust analysis of interdependencies, thereby increasing confidence in the results obtained.

In summary, these equations represent different ways to calculate the correlation between FDI (u) and the economic activity index (v) based on various models and statistical approaches, depending on the chosen parameters. The selection of the appropriate equation and parameters is subject to the specific objectives of the analysis and the nature of the data in question. Each provides a unique perspective on the relationship between these two economic variables.

5. DATA ANALYSIS

The descriptive analysis of the data is based on the IBC-Br and FDI deflated by the real exchange rate index, along with the chronological presentation of the Presidents of the Central Bank of Brazil. The period of analysis was monthly, spanning February 2003 to December 2022, with data obtained directly from the Central Bank of Brazil's API (2023). Graph 1 presents graphs (Absolute Values and log returns) of the variables studied.

It is noteworthy that during the presidents of the Central Bank of Brazil (2003 to 2022), the behavior of FDI and the IBC-Br varied significantly. During Henrique Meirelles' tenure (2003-2010), Brazil experienced a period of robust economic growth and macroeconomic stability. The level graph shows consistent growth of the IBC-Br, reflecting market confidence and stable economic policies. FDI also showed significant growth, especially between 2006 and 2008. However, the global financial crisis of 2008 resulted in an abrupt decline in the IBC-Br and a slight slowdown in FDI. In the log return graph, volatility increased during the crisis but gradually recovered until the end of the Meirelles term.

Under Alexandre Tombini (2011-2016), the economic scenario became more challenging. The level graph indicates a slowdown in IBC-Br growth with periods of stagnation and decline, especially during the 2014-2016 recession. FDI shows significant fluctuations and a downward trend, reflecting economic and political uncertainty. In the log return graph, high volatility is observed in both the IBC-Br and FDI, highlighting the impact of fiscal and economic crises during this period.

During Ilan Goldfajn's tenure (2016-2019), Brazil began a phase of economic recovery. Structural reforms, such as labor reforms and spending caps, were well received by the markets. The level graph shows the stabilization and slight recovery of IBC-Br. Although FDI stabilized, it did not return to the previous levels, reflecting investor caution. In the log return graph, volatility decreased, but economic challenges persisted, preventing a more robust recovery.

Under the management of Roberto Campos Neto (2019 to present), Brazil faced additional challenges during the COVID-19 pandemic. In the level graph, IBC-Br shows signs of stabilization and recovery despite the severe impact of the pandemic. Although initially impacted, FDI continued to flow, reflecting confidence in the implementation of pro-market policies. In the log return graph, high volatility was observed during the pandemic, followed by a recovery trend, highlighting the resilience of the Brazilian economy and the effective measures adopted to mitigate adverse effects.

In summary, the behavior of FDI and the IBC-Br varied significantly during the presidents of the Central Bank of Brazil (2003-2022). The implemented economic policies, global economic events, and domestic conditions directly influenced these indicators, with periods of growth and slowdown reflecting the complexity and dynamics of the Brazilian economy. The graph illustrates how different economic contexts and policy approaches have affected investment flows and economic activity over the years.

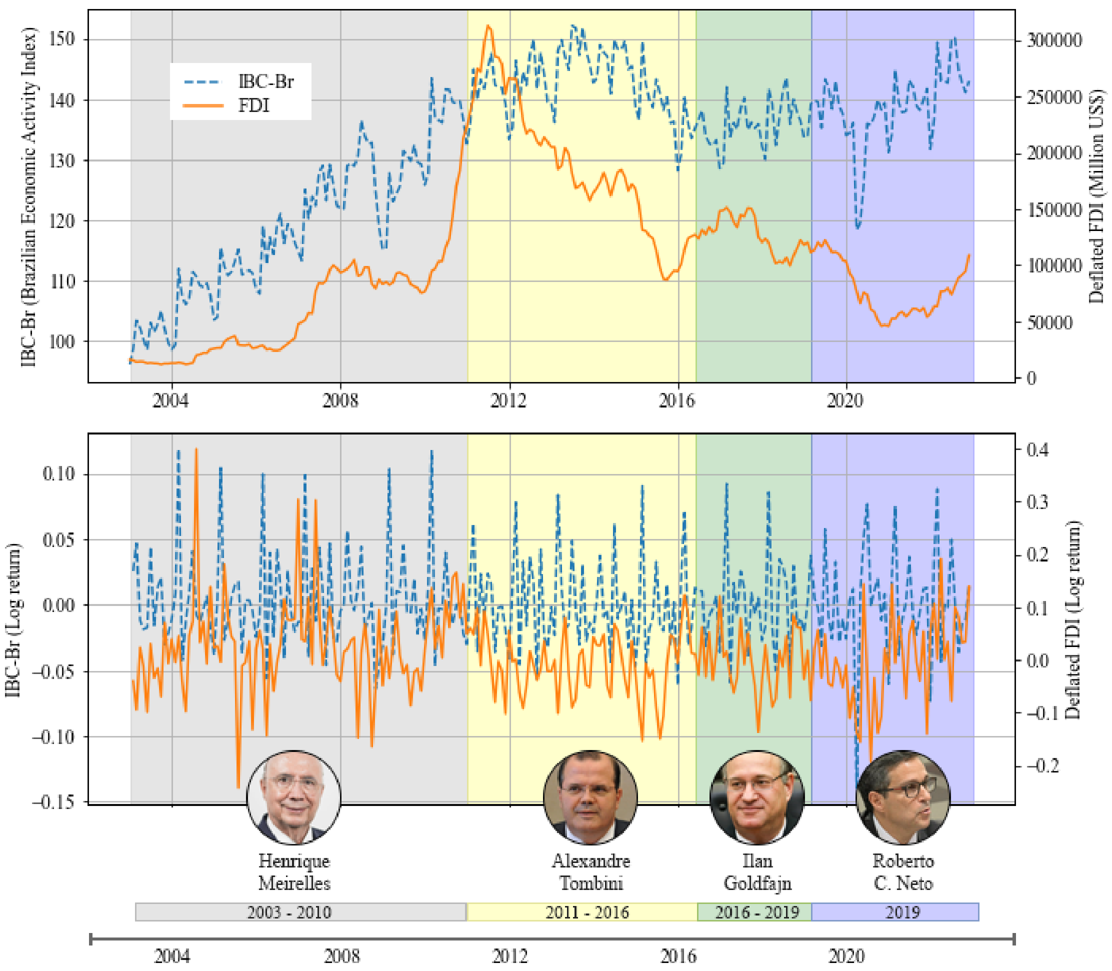
In conclusion, Table 1 presents a statistical summary of the data for the IBC-Br and FDI deflated by the real exchange rate index for the period from February 2003 to December 2022. This table includes values for the mean, standard deviation, minimum, maximum, and 25th, 50th (median), and 75th percentiles for both the absolute values and log returns of these variables.

The IBC-Br data show a mean of 131.57 with a standard deviation of 13.56, indicating moderate variation in economic activity. The minimum value is 98.58, and the maximum is 152.13, reflecting the extremes of economic growth and contraction. The median of 135.99 suggests a relatively stable economic performance.

For deflated FDI, the mean was approximately $107,425 million, with a standard deviation of approximately $69,324 million, indicating high variability. The minimum value was approximately $11,774 million, and the maximum was around $312,720 million, showing large fluctuations in investment flows.

The log-returns of the IBC-Br and FDI provide information on volatility over time. The mean log-return for IBC-Br was 0.00166, with a standard deviation of 0.03650, while for FDI, the mean log-return was 0.00801, with a standard deviation of 0.07998, indicating greater variability in FDIs.

Graph 1: Graphs of the IBC-Br and FDI in absolute values and log-return (February 2003 to 2022).



Source: Authors´ creation using BACEN data [15].

Table 1: Data Summary of the IBC-Br and of the FDI deflated by the real exchange rate for the monthly period from February 2003 to December 2022.

	IBC-Br	Deflated FDI	IBC-Br (Log return)	FDI (Log return)
Samples	239	239	239	239
Mean	131.57054	107,425.30628	0.00166	0.00801
Standard Deviation	13.56458	69,324.48635	0.03650	0.07998
Minimum	98.58000	11,773.96433	-0.13923	-0.24154
25%	123.16500	57,048.14653	-0.02042	-0.03594
50%	135.99000	96,758.11072	-0.00256	0.00831
75%	141.62000	144,254.21640	0.01649	0.04935
Maximum	152.13000	312,720.20723	0.11867	0.39897

Source: Own authorship using Python version 3.11 with BACEN data [15].

6. RESULTS AND DISCUSSION

6.1. GRANGER CAUSALITY

In Table 2, the Granger causality test results are applied to the variables IBC-Br and FDI, as well as their log returns. The results suggest that there is a significant causal relationship between FDI and IBC-Br when we consider the absolute values and short periods (one and two lags). However, this causality is only significant with FDI causing IBC-Br, indicating that FDI can influence IBC-Br in the short term. However, IBC-Br does not seem to have a significant causal effect on FDI.

Regarding the variables in percentage variation (log return), it was not possible to identify any causal relationship (in both directions) for any lag period. A possible interpretation of this result is that the volatilities or even the shocks (more evident when the trend of a series is removed, a fact that usually happens with transformation into log-return) of FDI and IBC-Br do not present direct linear causal relationships, unlike their absolute values or at level (more explicit in long-term relationships).

Table 2: Granger causality test results for the variables IBC-Br, FDI, and their respective log-returns (growth rate or percentage variations).

Lags	Absolute Values	F-Statistic	P-Value
1	FDI causing IBC-Br	7.30488***	0.00738
2	FDI causing IBC-Br	2.55086*	0.08020
3	FDI causing IBC-Br	1.41255	0.23986
4	FDI causing IBC-Br	1.10641	0.35430
5	FDI causing IBC-Br	0.91801	0.47006
6	FDI causing IBC-Br	0.86863	0.51873
7	FDI causing IBC-Br	0.87405	0.52780
8	FDI causing IBC-Br	0.75932	0.63901
9	FDI causing IBC-Br	0.68878	0.71869
10	FDI causing IBC-Br	0.70506	0.71919
1	IBC-Br causing FDI	0.07803	0.78023
2	IBC-Br causing FDI	0.44294	0.64269
3	IBC-Br causing FDI	0.46363	0.70795
4	IBC-Br causing FDI	0.57546	0.68072
5	IBC-Br causing FDI	0.72784	0.60321
6	IBC-Br causing FDI	0.86769	0.51943
7	IBC-Br causing FDI	0.86454	0.53540
8	IBC-Br causing FDI	0.78231	0.61873
9	IBC-Br causing FDI	0.68161	0.72515
10	IBC-Br causing FDI	0.69003	0.73318

Lags	Percentage Values (Log-return)	F-Statístic	P-Value
1	Log Return (FDI causing IBC-Br)	1.05199	0.30610
2	Log Return (FDI causing IBC-Br)	1.30575	0.27295
3	Log Return (FDI causing IBC-Br)	1.08995	0.35413
4	Log Return (FDI causing IBC-Br)	0.90886	0.45951
5	Log Return (FDI causing IBC-Br)	0.80918	0.54418
6	Log Return (FDI causing IBC-Br)	0.81750	0.55735
7	Log Return (FDI causing IBC-Br)	0.77838	0.60607
8	Log Return (FDI causing IBC-Br)	0.90128	0.51626
9	Log Return (FDI causing IBC-Br)	1.03750	0.41120
10	Log Return (FDI causing IBC-Br)	1.24520	0.26397
1	Log Return (IBC-Br causing FDI)	0.51086	0.47547

2	Log Return (IBC-Br causing FDI)	0.34610	0.70781
3	Log Return (IBC-Br causing FDI)	0.41194	0.74458
4	Log Return (IBC-Br causing FDI)	0.39801	0.80997
5	Log Return (IBC-Br causing FDI)	0.27580	0.92606
6	Log Return (IBC-Br causing FDI)	0.33348	0.91879
7	Log Return (IBC-Br causing FDI)	0.61509	0.74318
8	Log Return (IBC-Br causing FDI)	0.66999	0.71765
9	Log Return (IBC-Br causing FDI)	0.60013	0.79616
10	Log Return (IBC-Br causing FDI)	0.52341	0.87257

*** p<0.01, ** p<0.05, * p<0,01.

Source: Own authorship using Python version 3.11 with BACEN data [15].

It should be noted that these results are based on statistical analyses, and causality does not necessarily imply a direct causal relationship in the real world. Other factors that statistical models do not capture may play a role in the relationship between these variables. Therefore, these results should be interpreted with caution and considered part of a broader analysis.

6.2. STATIC COPULA FUNCTIONS

In sections 4.2 and 4.3, the results of the Copula Functions (Gaussian, Student's T, Gumbel, Frank, and Clayton) are presented in both static and dynamic forms. The static analysis involves modeling the dependency considering the entire data period (February 2003 to December 2022), whereas the dynamic analysis incorporates a more granular approach, using steps and sliding windows over time (window with 60 samples and 6 steps). This allowed us to assess how the dependency between the variables was for the entire period besides their evolution and variation over time, identifying possible changes in the relationships (a more comprehensive and precise understanding of the interactions between the variables under analysis).

Table 3: Results of the Gaussian, Student's t, Clayton, Gumbel, and Frank Copulas for the IBC-Br and IED sets in absolute values and percentage variations (Log-return).

Variables	Copula	Parameter	AIC	BIC	Kendall's Tau
IBC-Br and IED (absolute values)	Gaussian Copula	0.73195	-174.89573	-171.41926	0.52278
	t-Student Copula	0.73813	-175.81369	-168.86076	0.52858
	Clayton Copula	2.17068	-214.95441	-211.47795	0.52046
	Gumbel Copula	1.84425	-126.50989	-123.03343	0.45777
	Frank Copula	6.43846	-174.34010	-170.86364	0.53631
IBC-Br and IED (Log-return)	Gaussian Copula	0.03211	1.77262	5.24908	0.02045
	t-Student Copula	0.03214	3.79871	10.75163	0.02046
	Clayton Copula	0.05440	1.45619	4.93265	0.02648
	Gumbel Copula	1.01076	1.92800	5.40446	0.01064
	Frank Copula	0.21957	1.68903	5.16549	0.02439

Source: Own authorship using Python version 3.11 with BACEN data [15].

Table 3 summarizes the results of modeling the dependency between the IBC-Br and FDI using different copula functions. In this analysis, both the absolute values and log returns (percentage variations) of the variables were considered. A fundamental point to highlight is that the Clayton Copula has the smallest AIC and BIC values for representations of both the IBC-Br and FDI variables (absolute values and log returns).

The Clayton Copula is a model that assumes heavy-tail dependence, implying that extreme events in the time series of IBC-Br and FDI are more correlated than predicted by a standard linear normal model. In other words, atypical events or extreme movements in one variable are more sharply related to extreme movements in the other variable than expected in a normal relationship. This finding is crucial for understanding how extreme events can impact the joint dynamics of these variables and has significant implications for risk management and economic decision-making.

As a validation of Table 3, Graph 2 presents the analysis of the relationships between the IBC-Br and FDI using different copula functions, considering both absolute values and log returns. In the graphs of the original data (absolute values), a clear positive correlation between IBC-Br and FDI is observed, with a significant dispersion of points. The extreme values of the data show the maintenance of this relationship, reflecting periods of economic growth and crises. Among the various copulas analyzed, the Clayton copula stands out for its adherence to the original data and accurately captures the dependence in the lower tails. This means that the Clayton copula is particularly effective in modeling situations where both indicators are at low levels, reflecting periods of economic crisis or slowdowns. The ability of this copula to capture the concentration of points in the low values of IBC-Br and FDI demonstrates its usefulness in adverse economic scenarios, offering a detailed view of how foreign investments and economic activity react in difficult times.

For log returns, the graph shows more dispersion and a less visible relationship between IBC-Br and FDI, indicating that the correlation between the percentage changes of these series is less pronounced. The extreme values of log returns do not exhibit a clear dependence. The Gaussian and t-Student copulas simulate log returns, but the dispersion of points is greater, reflecting the lower correlation in the original data. Again, the t-Student copula better captures the tails, but due to the lower correlation in the log returns, this capture is less pronounced. The Clayton, Gumbel, and Frank copulas continue to show the relationship between the IBC-Br and FDI, but with less concentration in the extremes compared to absolute values. The Clayton copula shows dependence in the lower tail and Gumbel in the upper tail, but both are less pronounced. The Frank copula maintains symmetric dependence with less emphasis on extremes.

Therefore, the analysis of the graphs reveals that the relationship between IBC-Br and FDI is more evident in absolute values, with a strong dependence on the extremes captured by the Clayton copula. In log returns, the relationship is less visible, reflecting a lower correlation between the percentage changes in these variables. Copulas help capture different aspects of this dependence, highlighting the differences between the upper and lower tails. In particular, the Clayton copula stands out for its ability to adhere to the original data, especially in scenarios of low values, offering an in-depth view of economic relationships during periods of adversity.

6.3. DYNAMIC COPULA FUNCTION

The results of the Dynamic Copulas (Gaussian, t-Student, Clayton, Gumbel, and Frank) for the IBC-Br and FDI datasets in absolute values (steps of 6 months with windows of 60 months) and percentage variation (Log-return) are presented in Table 3. Kendall's tau (a non-parametric correlation measure calculated based on the estimated coefficients of the copulas) varies over time for each copula. This measure reveals whether the dependence is positive, negative, or near zero. Therefore, Kendall's tau movements indicate changes in the direction and intensity of dependence.

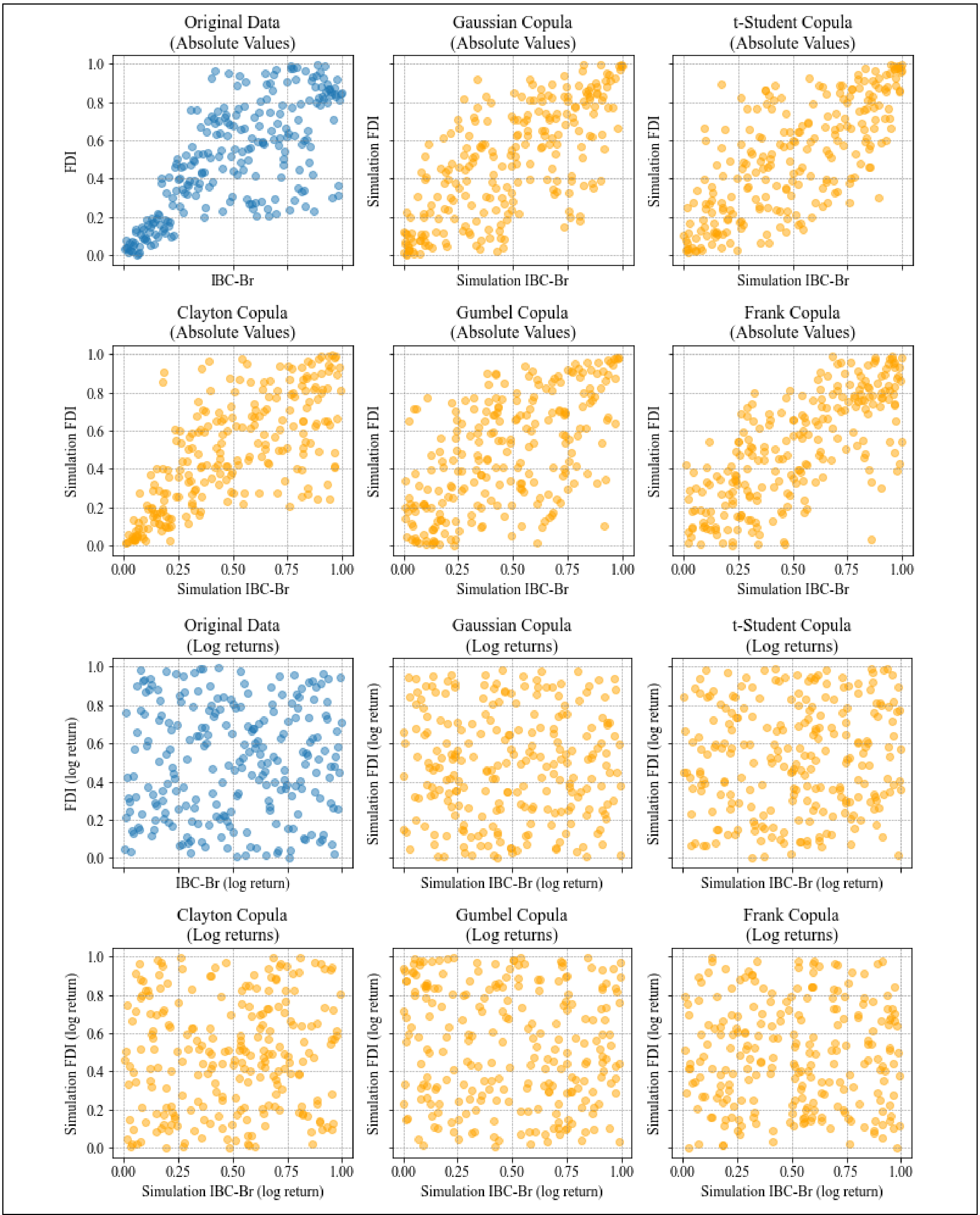
During Henrique Meirelles' term (01/01/2003–31/12/2010), the relationship between the variables in absolute values remained positive and strong throughout the period, while for percentage changes, there was neutrality with tau values close to zero. This period includes the global financial crisis of 2008, when the positive dependence between FDI and IBC-Br remained robust, suggesting the resilience of the Brazilian economy to external shocks. Other significant events include the commodity boom that boosted Brazil's economic growth and economic stabilization following the implementation of the Real Plan.

Alexandre Tombini (01/01/2011 - 07/06/2016) faced a sharp slowdown in the non-linear relationship between foreign investment and the economic activity index in levels, as well as the continued neutrality of the more volatile relationship of the variables (log-returns). This period includes significant events, such as the European sovereign debt crisis, which negatively affected global investment flows and the political crisis that culminated in the impeachment of President Dilma Rousseff in 2016. The observed decline in dependence can be attributed to political and economic instability, reflecting reduced external investor confidence in Brazil.

Under Ilan Goldfajn's leadership (09/06/2016–27/02/2019), there was a recovery in the positive relationship between foreign investment and economic activity, as well as a decrease in their more volatile relationship (log-returns). However, by the end of his term, it was already possible to observe the return of the relationships to the initial or near-initial levels. This recovery period can be associated with improved economic policies and a more stable political environment following the impeachment. During this time, Brazil also faced challenges, such as the truck driver strike in 2018, which significantly impacted the economy.

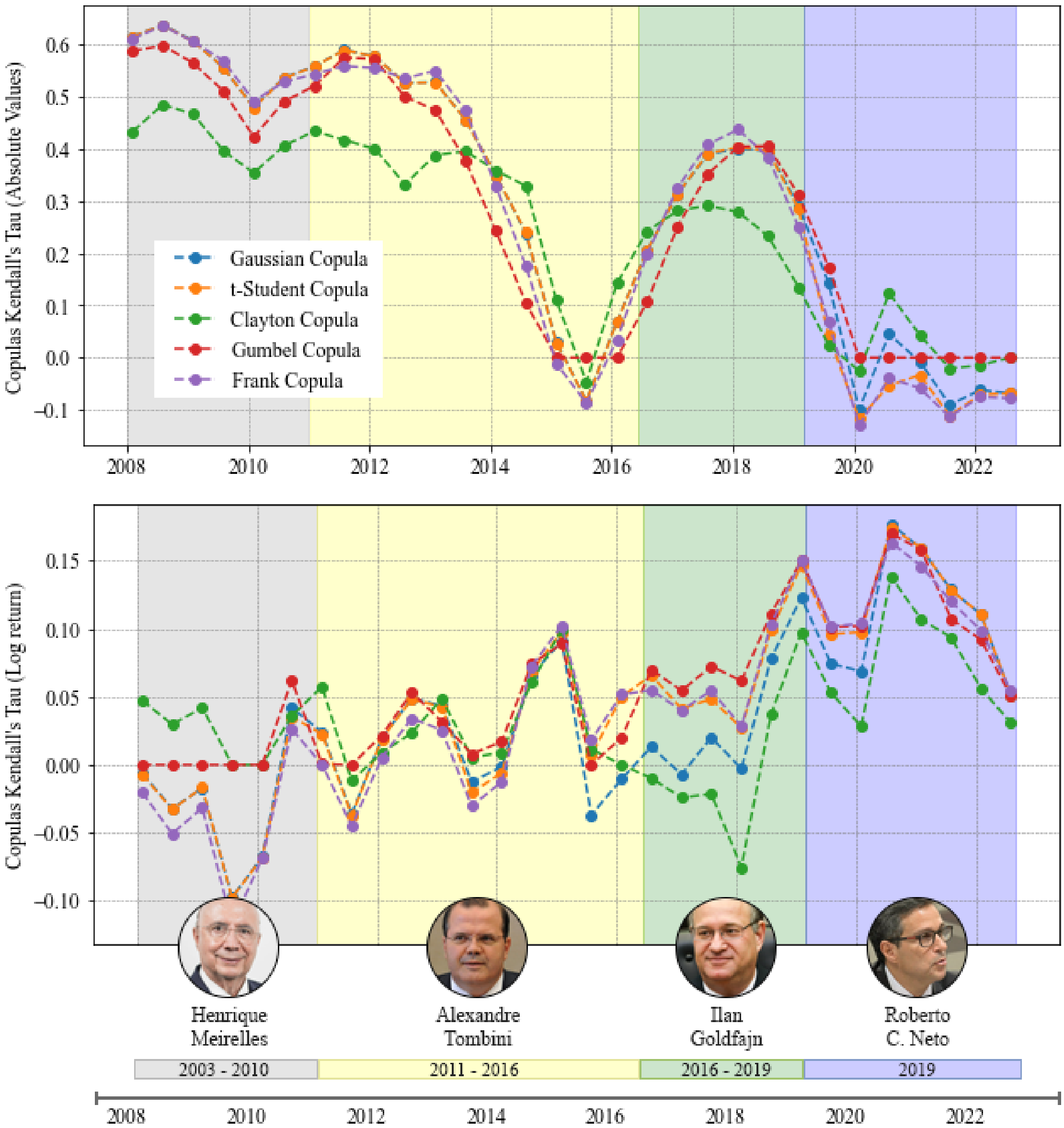
Roberto Campos Neto (28/02/2019 - up to the final date of analysis in this study) faced many reflections from previous mandates (60-month window), maintaining a neutral (close to zero) relationship between FDI and the IBC-Br for both absolute values and log returns. This period includes the COVID-19 crisis in 2020, a global event that brought about great economic uncertainty. Additionally, events such as the trade war between the United States and China and fluctuations in commodity prices have also influenced the Brazilian economy. The neutrality observed during this period can be attributed to global uncertainties and the diverse economic responses to this unprecedented crisis.

Graph 2: Graphs of the original and simulated data based on the estimated copulas (Gaussian, Student's t, Clayton, Gumbel, and Frank) of IBC-Br and deflated IED.



Source: Own authorship using Python version 3.11 with BACEN data [15].

Graph 3: Graphs of the Dynamic Copulas (Gaussian, t-Student, Clayton, Gumbel, and Frank with a window of 60 months and step of 6 months) for IBC-Br and deflated FDI in absolute values.



Source: Own authorship using Python version 3.11 with BACEN data [15].

Comparing the dynamic and static analyses of the dependency between IBC-Br and FDI, we observe some important differences and similarities. In the dynamic analysis, we notice that the relationship between these variables can change substantially in response to economic and political shocks such as the 2008 financial crisis, the European sovereign debt crisis, Dilma Rousseff's impeachment, and the COVID-19 pandemic. For instance, during Henrique Meirelles' term, the positive dependence remained robust, suggesting the resilience of the Brazilian economy to external shocks, while during the terms of Alexandre Tombini and Roberto Campos Neto, we observed neutrality or a decline in dependence, reflecting political and economic instability.

In contrast, the static analysis with the Clayton Copula reveals that regardless of the specific period, there is a heavy tail dependence between the series, indicating that extreme events in one of the variables tend to be associated with extreme movements in the other. This statistical finding is consistent over time and highlights the importance of considering extreme events in risk management and economic policy formulation. Therefore, while dynamic analysis provides a detailed view of how the relationship between IBC-Br and FDI evolves in response to different events, static analysis with the Clayton Copula offers a deep understanding of how extreme events consistently impact the dependency between these variables.

7. FINAL CONSIDERATIONS

The relationship between FDI and the IBC-Br is complex and dynamic over the analyzed period from February 2003 to December 2022. This analysis demonstrates the importance of FDI as a potential influencer of Brazilian economic activity in the short term, although this influence diminishes when considering longer periods. Conversely, IBC-Br does not exert similar causal reciprocity on FDI. This was evidenced by the application of the Granger causality test, which showed a significant causal relationship between FDI and IBC-Br in absolute values but not in the opposite direction.

The use of structural dependency functions, particularly copulas, provides a deeper understanding of the nature of this relationship. The Clayton Copula stood out as the most suitable model for characterizing the dependency between the two variables, both for absolute values and log returns. This suggests that, in scenarios of extreme events, especially negative ones, both time series (IBC-Br and FDI) move together more sharply than in a normal linear relationship. This understanding is critical for adverse scenario assessments and decision-making aimed at risk management.

The examination of copula parameters over time reinforces the idea of a mutable and adaptable relationship. Periods of positive dependency were identified, such as in early 2007, and notable transitions were identified, such as in 2015, when a shift to neutral dependency was observed. These fluctuations indicate the evolving nature of the relationship between FDI and the IBC-Br. The analysis of the series' log returns reiterated the fluid dynamics of the relationship, showing dependencies that varied over the years but still demonstrated significant interaction between the variables.

The terms of the Central Bank presidents during the analyzed period also played a significant role in the evolution of this relationship. During Henrique Meirelles' term (2003-2010), the positive dependency between FDI and IBC-Br remained robust, suggesting the resilience of the Brazilian economy to external shocks such as the 2008 global financial crisis. Under Alexandre Tombini's leadership (2011-2016), there was a sharp slowdown in this relationship, reflecting political and economic instability, including the European sovereign debt crisis and Dilma Rousseff's impeachment. During Ilan Goldfajn's term (2016-2018), there was a recovery in the positive relationship associated with improvements in economic policies and a more stable political environment. According to Roberto Campos Neto (2019 to the present work), this relationship remained neutral, reflecting the global uncertainties brought about by the COVID-19 pandemic and other international events.

In conclusion, this study reveals the complex and variable relationship between foreign investment and Brazilian economic activity. The dynamics over time indicate that economic policies and investment strategies must be constantly reassessed based on the latest interactions between FDI and IBC-Br. This understanding is essential for academics, policymakers, investors, and managers seeking to understand and adapt to changes in the economic landscape. Future studies could explore the influence of other factors, such as fiscal and trade policies, as well as the impact of global events on the relationship between FDI and the IBC-Br. Additionally, the analysis of new econometric methodologies and the inclusion of additional variables could provide a more comprehensive and accurate understanding of this relationship.

CONFLICT OF INTEREST

None.

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APPENDIX A: RESULTS OF THE DYNAMIC COPULAS (GAUSSIAN, T-STUDENT, CLAYTON, GUMBEL, AND FRANK) FOR THE IBC-BR AND FDI SETS IN ABSOLUTE VALUES (WINDOWS = 60 MONTHS; STEPS = 6 MONTHS).

Coefficients	Date	Gaussian Copula	t-Student Copula	Clayton Copula	Gumbel Copula	Frank Copula
Kendall's Tau of the copulas (Absolute values)	02/01/2008	0,61408	0,61416	0,43287	0,58643	0,61023
	08/01/2008	0,63649	0,63605	0,48381	0,59721	0,63454
	02/01/2009	0,60638	0,60489	0,46581	0,56331	0,60478
	08/01/2009	0,55462	0,55513	0,39582	0,51006	0,56797
	02/01/2010	0,47907	0,47789	0,35318	0,42233	0,49101
	08/01/2010	0,53528	0,53455	0,40506	0,48992	0,52707
	02/01/2011	0,55750	0,55692	0,43321	0,51956	0,54259
	08/01/2011	0,58857	0,58828	0,41613	0,57421	0,55806
	02/01/2012	0,57621	0,57606	0,39943	0,57137	0,55422
	08/01/2012	0,52582	0,52526	0,33180	0,49881	0,53414
	02/01/2013	0,52706	0,52642	0,38759	0,47363	0,54920
	08/01/2013	0,45536	0,45499	0,39494	0,37696	0,47316
	02/01/2014	0,34879	0,34848	0,35814	0,24231	0,32793
	08/01/2014	0,23751	0,23942	0,32858	0,10421	0,17488
	02/01/2015	0,02961	0,02838	0,10976	0,00000	-0,01130
	08/01/2015	-0,08297	-0,08294	-0,04670	0,00000	-0,08701
	02/01/2016	0,06945	0,06865	0,14220	0,00002	0,03406
	08/01/2016	0,20545	0,20538	0,24160	0,10667	0,19803
	02/01/2017	0,31317	0,31303	0,28104	0,25119	0,32570
	08/01/2017	0,38871	0,39028	0,29085	0,35085	0,40744
	02/01/2018	0,40046	0,40309	0,27883	0,40344	0,43647
	08/01/2018	0,39575	0,39568	0,23260	0,40455	0,38374
	02/01/2019	0,29032	0,28467	0,13492	0,31150	0,24940
	08/01/2019	0,14265	0,04315	0,02411	0,17403	0,06908
	02/01/2020	-0,09859	-0,11704	-0,02538	0,00000	-0,12942
	08/01/2020	0,04493	-0,05298	0,12295	0,00000	-0,03899
	02/01/2021	-0,00714	-0,03314	0,04373	0,00000	-0,05821
	08/01/2021	-0,08989	-0,11090	-0,02056	0,00000	-0,11299
	02/01/2022	-0,06190	-0,07059	-0,01475	0,00000	-0,07458
	08/01/2022	-0,06829	-0,06843	0,00000	0,00000	-0,07742

Source: Own authorship using Python version 3.11 with BACEN data [15].

APPENDIX B: RESULTS OF THE DYNAMIC COPULAS (GAUSSIAN, T-STUDENT, CLAYTON, GUMBEL, AND FRANK) FOR THE IBC-BR AND FDI SETS IN PERCENTAGE VARIATION (WINDOWS = 60 MONTHS; STEPS = 6 MONTHS).

Coefficients	Date	Gaussian Copula	t-Student Copula	Clayton Copula	Gumbel Copula	Frank Copula
Kendall's Tau of the copulas (Log return)	02/01/2008	-0,00623	-0,00702	0,04730	0,00000	-0,02034
	08/01/2008	-0,03244	-0,03311	0,02953	0,00000	-0,05085
	02/01/2009	-0,01714	-0,01668	0,04196	0,00000	-0,03164
	08/01/2009	-0,09776	-0,09801	0,00000	0,00000	-0,11751
	02/01/2010	-0,06792	-0,06816	0,00000	0,00000	-0,06893
	08/01/2010	0,04161	0,03507	0,03602	0,06193	0,02598
	02/01/2011	0,02388	0,02262	0,05723	0,00053	0,00002
	08/01/2011	-0,03629	-0,03729	-0,01179	0,00000	-0,04520
	02/01/2012	0,01958	0,01913	0,00896	0,02146	0,00542
	08/01/2012	0,04873	0,04825	0,02332	0,05329	0,03404
	02/01/2013	0,04336	0,04260	0,04869	0,03142	0,02526
	08/01/2013	-0,01255	-0,02060	0,00520	0,00775	-0,03051
	02/01/2014	-0,00160	-0,00590	0,00909	0,01699	-0,01243
	08/01/2014	0,06595	0,06855	0,06033	0,07404	0,07243
	02/01/2015	0,09101	0,09750	0,09912	0,08886	0,10198
	08/01/2015	-0,03697	0,00686	0,01140	0,00001	0,01827
	02/01/2016	-0,01065	0,04997	0,00000	0,01966	0,05152
	08/01/2016	0,01322	0,06574	-0,00998	0,07005	0,05480
	02/01/2017	-0,00730	0,04104	-0,02428	0,05438	0,03985
	08/01/2017	0,01994	0,04846	-0,02146	0,07242	0,05458
	02/01/2018	-0,00296	0,02685	-0,07557	0,06164	0,02819
	08/01/2018	0,07805	0,09907	0,03721	0,11119	0,10335
	02/01/2019	0,12298	0,14641	0,09734	0,14990	0,15013
	08/01/2019	0,07422	0,09595	0,05278	0,10122	0,10175
	02/01/2020	0,06868	0,09747	0,02828	0,10198	0,10383
	08/01/2020	0,17703	0,17454	0,13821	0,17008	0,16349
	02/01/2021	0,15886	0,15951	0,10686	0,15810	0,14608
	08/01/2021	0,12879	0,12841	0,09369	0,10736	0,12010
	02/01/2022	0,11037	0,11071	0,05616	0,09190	0,09878
	08/01/2022	0,05129	0,05142	0,03075	0,05056	0,05427

Source: Own authorship using Python version 3.11 with BACEN data [15].